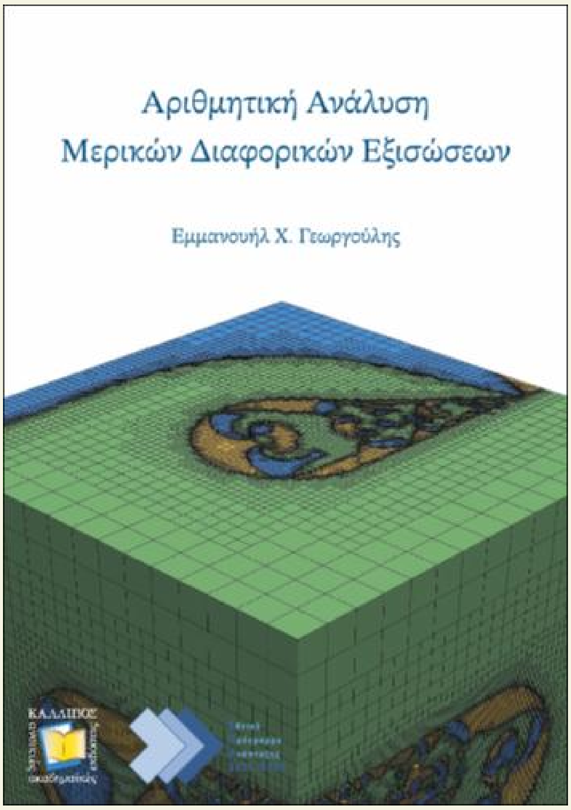

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Abstract

This book aspires to provide an extensive introduction to the Numerical Analysis of Partial Differential Equation (PDE) problems and to serve as a textbook for both advanced undergraduate and graduate courses in this topic. The presentation of two popular categories of methods is chosen: Finite Difference Methods (FDM) and Finite Element Methods (FEM). The volume contains a brief, and inevitably incomplete, review of basic PDE theory (Chapters 1 and 7) covering key concepts, properties and results, as well as some advanced topics in modern PDE theory, such as the weak form, Lebesgue and Sobolev function spaces, energy method etc. Chapter 2 gives a brief introduction to divided differences and presents the basic finite difference method for the one-dimensional second-order boundary value problem. We proceed by presenting finite difference methods for parabolic (Chapter 3), linear hyperbolic (Chapter 4), nonlinear hyperbolic (Chapter 5), and elliptic PDE problems (Chapter 6), with special attention to the study of consistency, stability and convergence of the methods, for which we prove basic error estimates. Next, we proceed with the presentation and analysis of finite element methods for linear elliptic problems (Chapter 8). We present FEM for parabolic problems in Chapter 9. In Chapter 10, an introduction to the so-called, a posteriori error analysis is given, which is then used to define algorithms for automatically adjusting the local "resolution" of FEM for elliptic and parabolic problems. Chapter 11 gives the basic principles and error analysis of the Discontinuous Galerkin Method for these problems. Finally, in Chapter 12, we present some advanced topics in FEM theory. In particular, some basic concepts are given for the definition of high-order finite elements, as well as a space-time FEM with time stepping defined via a discontinuous Galerkin method.

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