



METADATA

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Abstract

The first chapter is a reminder of basic notions from General Topology. In the second chapter quotient spaces are studied, a concise introduction to topological manifolds is given and the classical classification of compact surfaces is presented. In the third chapter topological groups and group actions on topological spaces are studied, with emphasis on the useful notion of even actions. The fourth chapter is an introduction to homotopy of maps, fundamental groups of topological spaces and homotopy equivalence. In the fifth chapter simply connected spaces, contractible spaces, deformation retractions and strong deformation retractions are introduced and the proof that topological groups have abelian fundamental groups is given. The sixth and seventh chapters introduce coverings, liftings of maps and homotopies, the monodromy action, universal covering spaces, hierarchies and automorphisms of coverings, even actions and normal coverings. The eighth chapter introduces Singular Homology. Homology

groups of topological spaces and homomorphisms induced by continuous maps are studied and the homotopy invariance of homology is proven. In the ninth chapter higher homotopy groups are introduced and the connection between homotopy and homology is presented via the Hurewicz Theorem. Also, using the Mayer-Vietoris sequence, the homology groups of spheres are calculated. In the tenth chapter, important theorems in Algebraic Topology are proven, namely Invariance of Dimension, the Hairy Ball Theorem and the Brouwer Fixed Point Theorem. The study of embeddings of spheres into spheres also establishes Invariance of Domain and the Jordan-Brouwer Theorem. The general Seifert-Van Kampen Theorem is stated and Grothendieck's proof of a special case is given. The fundamental groups of compact surfaces are calculated, using basic tools from Combinatorial Group Theory. Finally, a concise reference is made to foundational issues, like the word problem, the isomorphism problem and the homeomorphism problem.