METADATA

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Abstract

The book presents in detail the theory of Commutative Rings. First, the origins of Commutative Algebra are presented, and its connections to Number Theory, Algebraic Geometry and Invariant Theory are made. Next, the notion of the radical of an ideal as well as the extremely useful notion of localization with respect to a multiplicatively closed subset are given. Noetherian rings are defined, and Hilbert's Basis Theorem is proved. The notion of a primary decomposition is discussed. An introduction to monomial orderings and Groebner bases, a particularly important computational tool, is provided. For a ring R, the notion of an R-module is introduced, and a brief introduction to their Homological Algebra is given. The localization of R-modules is introduced as well, and properties of Noetherian and Artinian modules are considered. It is shown that every Artinian ring is Noetherian. The notion of integral ring extension is studied, with applications to number theory. Noether's Normalization Lemma and various forms of the equally fundamental Hilbert's Nullstellensatz are proved. Krull's dimension theory is presented. Hilbert's and Hilbert-Samuel functions are defined and their connection with dimension theory are studied. Valuation rings and Dedekind integral domains are presented. An introduction to the theory of completion is given. Each chapter contains a section with related concepts to Algebraic Geometry while most of them have a section with related code in Macaulay2 Symbolic Algebra computing package. Moreover, each chapter contains many examples and unsolved exercises and the book is suitable for independent study in undergraduate and graduate level.

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