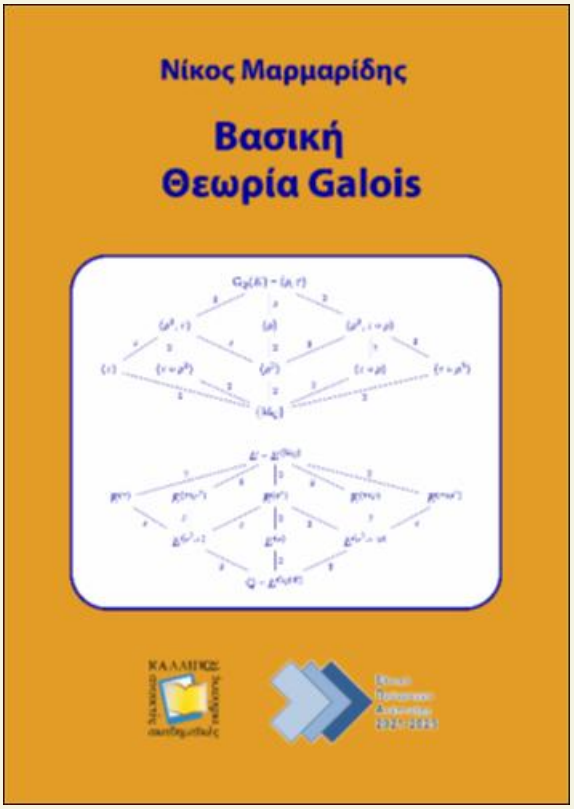

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Abstract

This book is a thorough presentation of Galois Theory. After recalling basic notions of ring theory, we prove Gauss Lemma for unique factorization domais. We also present and prove a generalization of Eisenstein's Criterion for integral domains. We study algebraic, separable and normal extensions of fields. We prove the existence of algebraicaly closed fields and of the algebraic hull of a field. We characterize Galois extensions via the theory of group actions on sets and we also study finite Galois extensions. We prove Artin's Lemma and present a procedure for determining fixed fields (relative to a subgroup of the Galois group) of simple finite field extensions. We study the field of rational functions as an extension of the field of symmetric rational functions. Using the theory of symmetric polynomials, we obtain the formulas describing the roots of the cubic and biquadratic polynomials. We prove the Fundamental Theorem of Galois Theory and the Fundamental Theorem of Algebra. We study extensively the finite fields. We give and prove necessary and sufficient conditions in order to be a complex number constructible by compass and lineal. We also give and prove necessary and sufficient conditions in order to be a regular n-gon constructible by compass and lineal. We study the splitting fields of cyclotomic polynomials and of polynomials of the form x^n-a. We prove Galois Theorem of the solvability of a polynomial by radicals. We determine the Galois groups of cubic and biquadratic polynomials. Using relative resolvents, we study the the Galois group of a polynomial. Finally we present and study the Weber resolvent which gives a sufficient and necessary condition in order to be solvable by radicals an irreducible polynomial of fifth degree with rational coefficients. The book contais many non trivial examples and over 365 exercises.

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