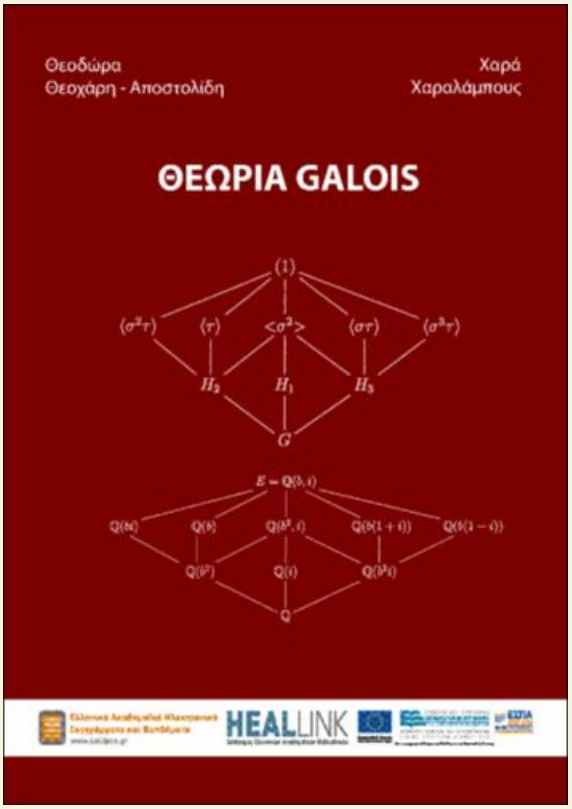


## METADATA

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**Abstract**

The book develops the Galois Theory of field extensions. The theory of polynomials and the determination of their roots is a subject widely used in all branches of mathematics and their applications. In this book, we develop the theory of polynomials over a field and the process of finding polynomial roots, based on the theory of field extensions, particularly finite-dimensional extensions. Automorphisms of fields play an extremely important role, and their extensive study precedes the proof of the Fundamental Theorem of Galois Theory. The theory of finite fields is developed, and methods for finding irreducible polynomials over such fields are presented. We also present applications of Galois Theory to solving polynomial equations over both fields of characteristic zero and finite fields. The classical unsolved problems of constructability with a straightedge and compass from antiquity are discussed. Sufficient and necessary conditions for the constructability of regular polygons are also provided. A proof of the Fundamental Theorem of Algebra is presented. Galois Theory culminates in solvability theory, offering complete information on when algebraic formulas exist for the roots of a polynomial, i.e., when a polynomial is solvable by radicals. Throughout the text, we highlight the role of permutation groups in the solvability of polynomials and provide numerous examples. In the appendix, we include the necessary material on Group Theory and Ring Theory. A brief historical overview is given at the beginning of the text. Exercises play a crucial role in reinforcing the material, and extensive hints are provided. An extensive bibliography is included at the end.

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