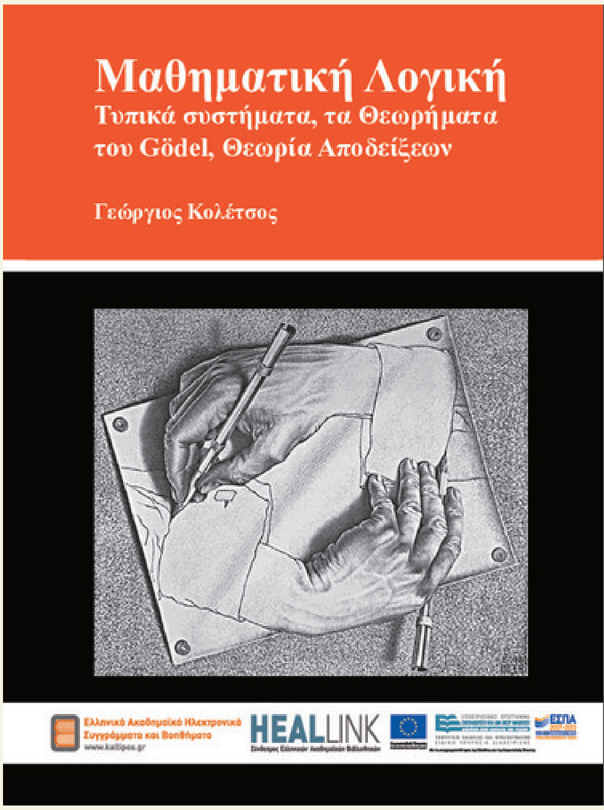

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Abstract

Mathematical Logic is closely connected with the foundational programs of Mathematics. The most well-known and coherent of these, Hilbert's program, introduced the concept of formal axiomatic systems and the issue of decidability, which are the two fundamental pillars of the sciences of Logic and Computer Science. The goal of this program was to prove the consistency of mathematical theories. Gödel, by demonstrating the impossibility of achieving this goal, introduced the primitive recursive functions as well as the ability of formal systems to reference themselves. This led Turing to formulate the model of computation and to prove the first results of undecidability. Recursive functions were introduced, clarifying the previously obscure landscape of computability. The outcome was the creation, at a theoretical level, of the science of Computer Science, while simultaneously making a decisive contribution to the foundation of mathematics and set theory.

The purpose of this book is to present all the classical results of logic which subsequently became essential in any serious study of the foundations of mathematics and theoretical computer science. The concept of formal proof systems, provability, the interpretation of formal theories, and completeness theorems are subjects of the book. Additionally, the study of recursive functions and the proof of Gödel's incompleteness theorem, which evolved into the undecidability results of Gödel, Turing, and Church, will be explored. Proof theory, which took its refined form from Gentzen, is concerned not only with what is proven but also with how it is proven. All the major results will be presented, such as the sequent calculus, natural deduction, and the famous cut-elimination theorem, which, through the isomorphism of proofs with programs, essentially constitute a mathematical study of the structure of programs in the field of computer science.

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