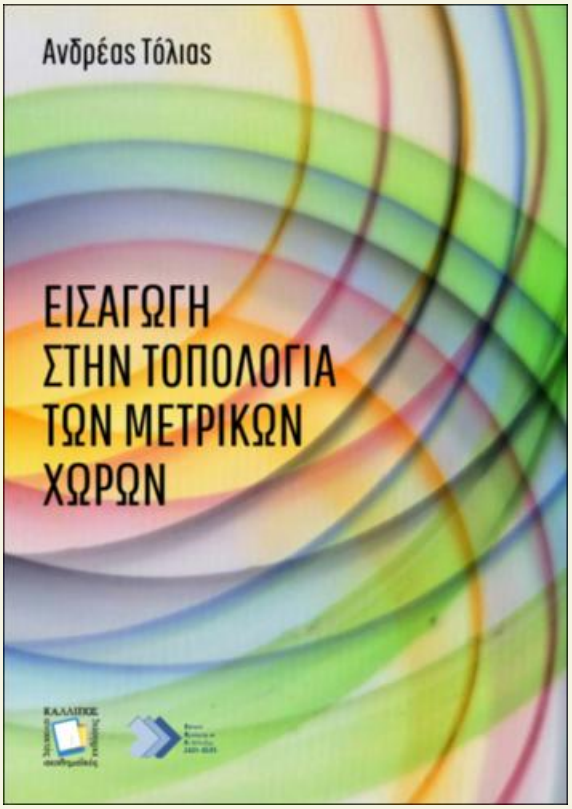

METADATA

Title: Introduction to Topology of Metric Spaces

Other Titles: -

Language: Greek

Authors: Tolias, A., Associate Professor, UOI

ISBN: 978-618-228-315-8

Subject: MATHEMATICS AND COMPUTER SCIENCE

Keywords: Metric space / Normed linear space / Open set / Interior of a set / Closed set

Bibliographic Reference: Tolias, A. (2025). Introduction to Topology of Metric Spaces [Undergraduate textbook]. Kallipos, Open Academic Editions. http://dx.doi.org/10.57713/kallipos-1065

Abstract

This book aims to provide a comprehensive introduction to the theory of metric spaces from both a theoretical and applied perspective. It is addressed to those who possess a basic background in Infinitesimal Calculus. In Chapter 1, the definition of a metric space is provided along with many examples, with the key feature element therein being the metrics induced by norms. In Chapter 2, the convergence of sequences in metric spaces is under study, whereas Chapter 3 provides a rigorous treatment of continuity and uniform continuity of functions between metric spaces. Chapter 4 covers the basic topological concepts in metric spaces (open set, interior of a set, closed set, closure of a set, derived set) and provides characterisations of the continuity of a function via topological arguments. Chapter 5 deals with homeomorphisms between metric spaces, equivalent metrics and the relatively open and relatively closed subsets of a subset in a metric space. Chapter 6 is concerned with the study of complete metric spaces and some basic relevant theorems. Chapters 7 and 8 are dedicated to the study of the notions of compactness and connectedness in metric spaces, respectively. Chapter 9 is concerned with function spaces and contains, among others, the Stone-Weierstrass Theorem. Chapter 10 addresses additional topics that have not been treated in the preceding chapters (completion of a metric space, the Cantor set, Tietze's Theorem for metric spaces, and oscillation of a function around a point). Finally, Chapter 11 provides the solutions to the exercises presented at the end of each of the previous chapters.

The Project is funded by the National Development Programme 2021-2025 of the Ministry of Education and Religious Affairs and implemented by the Special Account for Research Funds of the National Technical University of Athens and the Hellenic Academic Libraries Link.