
# METADATA

**Title:** Notes on Descriptive Set Theory

**Other Titles:** -

**Language:** Greek

**Authors:** Gregoriades, V., Assistant Professor, NTUA

**ISBN:** 978-618-228-224-3

**Subject:** MATHEMATICS AND COMPUTER SCIENCE

**Keywords:** Polish spaces / Baire space / Cantor space / Zero-dimensional spaces / Pointclass

**Bibliographic Reference:** Gregoriades, V. (2024). Notes on Descriptive Set Theory [Postgraduate textbook]. Kallipos, Open Academic Editions. http://dx.doi.org/10.57713/kallipos-963

**Abstract**

In the first chapter we briefly present the concepts, which are needed throughout the book. In the second chapter we present the theory of Polish spaces. Emphasis is put on the fundamental spaces of Baire and Cantor and on the concept of a tree. We study the Cantor-Bendixson Theorem, some universal properties of Polish spaces and the zero-dimensional Polish spaces. Further we present some topological characterizations of the Baire space and the Cantor space. The third chapter is concerned with the general theory of pointclasses in Polish spaces as well as the fundamental operators from logic and set theory. We introduce two of the most fundamental types of pointclasses: the Borel pointclasses of finite order and the projective pointclasses. Further related concepts are studied, such as the completeness with respect to a pointclass, the uniformization and the separation of sets. The fourth chapter is concerned with Borel sets and Borel-measurable functions. We study the Borel-isomorphism Theorem, the relation between sets and injective functions, and we prove the well-known representation of Borel sets as the continuous injective images of closed subsets of the Baire space. Finally, we present the Borel pointclasses of transfinite order and we prove their fundamental properties. In the fifth chapter we study deeper the pointclass of analytic sets. We give the proofs of some fundamental results about this pointclass: the Lusin Separation Theorem, the Susin Theorem, the Perfect Set Theorem, the Kunen-Martin Theorem and the theorem on the measurability of analytic sets. The sixth chapter is concerned with the deeper study of the pointclass of coanalytic sets and their applications in the uniformization of Borel sets. We present a new approach to the later with techniques from effective descriptive set theory. In the seventh chapter we present some elements from game theory and its applications in descriptive set theory. The eighth chapter is concerned with some applications from graph theory. The notes contain a large collection of exercises of varying difficulty. Hints and solutions are given in the appendix.

The Project is funded by the National Development Programme 2021-2025 of the Ministry of Education and Religious Affairs and implemented by the Special Account for Research Funds of the National Technical University of Athens and the Hellenic Academic Libraries Link.